$$\pi r = \pi a\left(1 - \frac{e^2}{2^2} - \frac{1^2.3.e^4}{2^2.4^2} - \frac{1^2.3^2.5.e^6}{2^2.4^2.6^2} - \right).$$

Solving by trial, $e=.895$, and $b=1.784$.

At a distance of z from the circular end of the pipe, the section is an ellipse, the semi-axes of which are found to be

$$\frac{hr-rz+az}{h} \text{ and } \frac{hr-rz+bz}{h}.$$

Hence the volume is

$$\int_0^h \pi\left(\frac{hr-rz+az}{h}\right)\left(\frac{hr-rz+bz}{h}\right)dz = \frac{\pi h}{6}(ar+br+2r^2+2ab) = 198.5\pi.$$

Hence the loss$=216\pi-198.5\pi=17.5\pi=55$ cubic inches.

MISCELLANEOUS.

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82. Proposed by A. H. BELL, Hillsboro, Ill.

Four spheres of equal radii$=r=5$, are in contact, and form a triangular pyramid. How large is the sphere that can be placed in the middle and be in contact with the four spheres.

Solution by J. W. YOUNG, Fellow in Mathematics, Cornell University, Ithaca, N. Y., and J. SCHEFFER, A. M., Hagerstown, Md.

Let A, B, C, B' (Fig. 1) be the centers of the four spheres. They evidently form the corners of a regular tetrahedron. Fig. 2 is a picture of a plane section of the pyramid of spheres, passed through the points $AB'L$, where L is the point of tangency of the two spheres (C, B).

From Fig. 1,

$$\left.\begin{array}{l} AN/AM=\sin 60^\circ \\ AN/AB'=\cos 60^\circ \end{array}\right\}.$$

$\therefore$ $AB'/AM=\tan 60^\circ=\sqrt{3}=\sec DAM$.

In Fig. 2, then, $\angle DAM$ is $\sec^{-1}\sqrt{3}$. It is clear that the required small sphere must have its center on DM and must touch both spheres (A, D). Let $\angle ADM=\theta$.

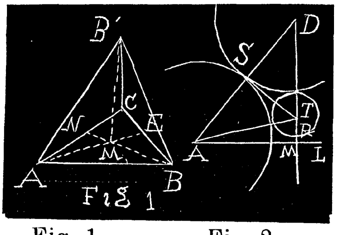

Fig. 1. Fig. 2.

Then $\sin\theta=1/\sqrt{3}$, $\cos\theta=\sqrt{\tfrac{2}{3}}$, $DT/r=\sqrt{\tfrac{3}{2}}$.

$\therefore$ $DT=(r/2)\sqrt{6}$.

$\therefore$ $RT=DT-r=(r/2)(\sqrt{6}-2)=$radius of small sphere.

$r=5$ gives $RT=1.1238$.

85. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Prove that at least one of the three sides of a rational right triangle must be divisible by 5.